SOLUTION TO UNSTEADY BOUNDARY-LAYER EQUATIONS

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In [1, 2] it has been shown that the laminar boundary-layer problems which are self-similar in steady flow are also self-similar in unsteady flow if the flow is impulsively started or accelerated with a step increase. A conclusion has been reached on the dominating effect of the body shape when compared to the acceleration of the body on the boundary-layer velocity and temperature profiles. In the present paper, we consider a number of non-self-similar unsteady boundary-layer problems. A change in variables is suggested for computations in order to increase the time step size by almost an order of magnitude. This considerably reduces the computational time. The basic relation is "encompassed" by the self-similar part of the problem. An analysis is made of the effect of various parameters (acceleration, pressure gradient, fluid composition, suction, dependence of viscosity on velocity) on skin-friction and heat flux to the wall.

1. Model for Local Similarity of Unsteady Boundary Layer. It is assumed that the external flow velocity can be expressed in the form $U_e = ax^{m(x)}t^{\alpha}$. Here and in what follows a, α are constants, x is the streamwise coordinate, y is the transverse coordinate, the index e refers to quantities in the external flow, m(x) is a given function, u is the streamwise velocity component, and v is the normal velocity component.

New coordinates are introduced for solving the problem:

$$\xi = x/t^{p(x)},\ \eta = y/t^{1/2},\ \tau = t,\ u = \overline{\Phi}(\xi,\ \eta,\ \tau)ax^{m(x)}t^{\alpha}, \tag{1}$$

where

$$p = (\alpha + 1)/(1 - m(x));\quad v = V(\xi,\ \eta,\ \tau)/t^{k(x)};\quad k(x) = 1/2.$$

It is known [3] that at the beginning of the motion vorticity of infinite strength appears in the neighborhood of the wall. Hence, computation at small values of time is carried out separately. The equation given in [4], for example, is solved:

$$\frac{d^2\widetilde{\Phi}}{d\eta^2} + \frac{1}{2}\eta\frac{d\widetilde{\Phi}}{d\eta} - \alpha\widetilde{\Phi} = 0,\ \widetilde{\Phi}(0) = 1,\ \widetilde{\Phi}(\infty) = 0,\ \widetilde{\Phi} = 1 - \overline{\Phi}. \tag{2}$$

Equation (2) is solved by the shooting technique. In terms of the variables (1) the system of boundary-layer equations has the form (the bar is dropped, $\Phi = \overline{\Phi}(\xi,\ \eta,\ \tau)$),

$$\begin{gathered}\alpha\Phi - \xi p\frac{\partial\Phi}{\partial\xi} - \eta k\frac{\partial\Phi}{\partial\eta} + \Phi\Big[a\xi^{m(x)}(1 - x\ln tp'(x))\frac{\partial\Phi}{\partial x} + \\ + \Phi a\xi^{m(x)-1}(m'(x)x\ln x + m(x))\Big] + a\xi^{m(x)-1}V(1 - x\ln tk'(x))\frac{\partial\Phi}{\partial\eta} = \\ = \alpha + a\xi^{m(x)-1}(m'(x)x\ln x + m(x)) + \frac{\partial^2\Phi}{\partial\eta^2} - \tau\frac{\partial\Phi}{\partial\tau}; \\ \Phi(m'(x)x\ln x + m(x)) + \xi(1 - x\ln tp'(x))\frac{\partial\Phi}{\partial\xi} + \frac{\partial V}{\partial\eta} = 0;\ \Phi(0,\xi) = \\ = 0, \xi > 0, \tau > 0;\ V(0,\xi) = w;\ \Phi(\infty,\xi) = 1;\ \Phi(\eta, 0) = 1;\ w = \text{const}.\end{gathered} \tag{3}$$

In order to solve system (3), an implicit difference scheme in ξ, the shooting method with iteration in η, and explicit difference scheme in τ are used. The quantities $(1 - x\ln tp'(x))$ and $(m'(x)x\ln x + m(x))$ are easily computed through ξ, η, τ.

An example of the computation of the nondimensional velocity profile is shown in Fig. 1 for $m(x) = \delta + \gamma x$, $\delta = 0.5$, $\gamma = -0.1$, $+0.1$. The curves for $\gamma = -0.1$ and $\gamma = 0.1$ coincide. Similarity solutions differ by the second sign from the solution to the complete system of equations. Curves 1, 2 correspond to $\xi = 0.05$, 3.1.

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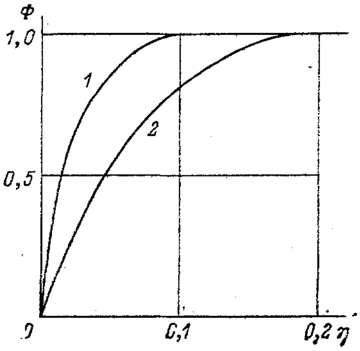

Fig. 1

The results of these computations make it possible to conclude about the usefulness of approximating realistic unsteady flows of the given type by locally similar flows.

2. Unsteady Compressible Boundary Layer for Various Speeds at the Outer Boundary. Assume $U_e = ct^{\alpha}x^m$, $U_e = (1 + ct)^b \alpha x^m$, m = const. Consider the situation when the fluid flow outside the boundary layer is adiabatic, the fluid is ideal and perfect. For the velocity distribution $U_e = ct^{\alpha}x^m$ the pressure at the outer boundary can be approximately computed with Cauchy—Lagrange equation

$$p \simeq -\rho_e\left[\frac{c\alpha}{m+1}t^{\alpha-1}x^{m+1} + \frac{c^2}{2}t^{2\alpha}x^{2m}\right] + p_0(t) = p_1 + p_0(t) = p_1 + \text{const}, \tag{4}$$

where p_0 is the free-stream pressure. Equation (4) is obtained by assuring that $\rho_e \approx$ const and $T_e \approx$ const. If the temperature at the outer boundary is high, e.g., T_e = 2500°K, p_0 = 10^5 N/m², c = 500, $\rho_e \approx 0.15$ kg/m³, then for $0 \leqslant \alpha \leqslant 1.5$, m = 0.5, $0.5 \cdot 10^2 \leqslant |p_1| \leqslant 0.3 \cdot 10^5$ N/m², i.e., $p_0 \gg p_1$ and the assumption is valid. The given model is discussed because we do not have the necessary information on the distribution of parameters at the outer boundary for unsteady flow. For compressible fluid, a binary mixture and air were considered. Viscosity μ and conductivity λ were computed using an equation given in [5]. The algorithm for the solution of the problem was the same as in Sec. 1. Since m(x) = const, the solution rapidly shifts along ξ to the asymptote (ξ ≈ 5).

Figure 2 shows nondimensional velocity profiles on the plate for a mixture of O, O_2 with concentrations of C_{O_2} = 0.7 and C_O = 0.3 (solid lines) and H_2, O_2 with concentration of C_{O_2} = 0.7 and C_{H_2} = 0.3 (crosses) at $U_e = 0.2 \cdot 10^5$ cm/sec, $T_e \approx$ 1500°K, T_w = 500°K, ξ = 0.025; 6.4 (curves 1, 2, respectively). Computed results showed that the relation between mass and heat transfer is the same as in the steady flow.

Figure 3 illustrates the variation in velocity profile on a flat plate in air as a function of suction. Here c = 500, $T_e \approx$ 2400°K, T_w = 500°K, $v_w = \omega t^{1/2}$, α = 1, 0.5; t = 0.5, 1; 2 sec, ω = 0, 2.5; 10, 15 (all curves coincide), ξ = 0.00625, 5.5 (curves 1, 2, respectively). The dashed line corresponds to ξ = 5.5, ω = 50, the dot-dash line is for T_w = 70°K, ξ = 5.5. It is seen that the effect of vertical velocity component is small and for appreciable reductions in skin-friction, a large mass flow of fluid through the wall is required. For 0 < m < 1, the behavior of the profiles is similar. Similar computation was made for $U_e = (1 + ct)^b \alpha x^m$.

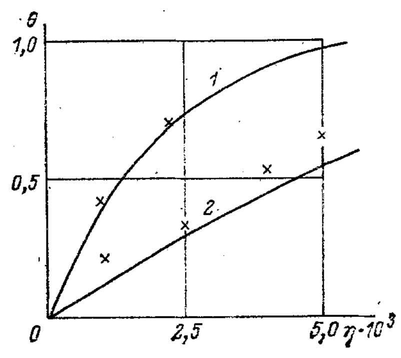

Fig. 2

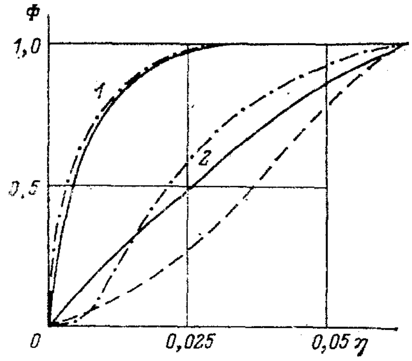

Fig. 3

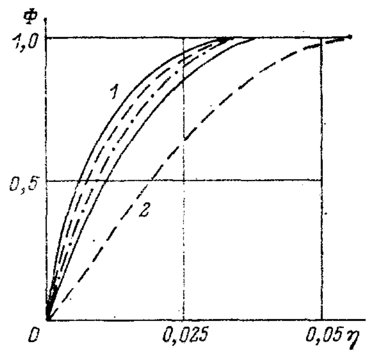

Fig. 4

In both cases, the introduction of new variables reduces computation time.

3. Unsteady Boundary Layer with External Flow Velocity $U_e = ct^{\alpha}x^m$ for Non-Newtonian Fluid. The inadequacy of theoretical and experimental data on the effect of the nonlinear characteristic of viscosity on the velocity and temperature boundary layer profiles is indicated in [6]. We studied the case when viscosity $\mu = c_1(\partial u/\partial y)^{k_1}$, conductivity $\lambda = c_2(T/T_0)^{k_2}$, c_1, k_1, c_2, k_2, and T_0 are given constants. In the case of incompressible flow started in pulses or with a sudden acceleration, the problem is self-similar and the number of variables drops from three to two. Computation is carried out as in [1, 2].

With increase in k_1, nondimensional velocity profiles become more flat when compared to the case of constant viscosity. This is illustrated in Fig. 4. Here the solid line refers to μ = const, $c = 500$, $m = 0.5$, $\alpha = 0.5$, $\xi = 0.02$; 3.6 (curves 1, 2 respectively), $k_1 = 0.5$ (dashed line), $k_1 = 0.7$ (dash—dot line). For compressible fluid it is not possible to reduce the order of the equation. However, after transforming the variables, it is possible to compute with large step sizes in τ. Viscosity is the dominating factor that determines the temperature profile in the non-Newtonian fluid with pulsed starting of the motion of the body. For the usual fluids, the temperature profile is always determined by heat conductivity.

The above computations showed the effectiveness of using the suggested coordinate system and made it possible to study the effect of individual parameters on the characteristics of the unsteady boundary layer.

LITERATURE CITED

1. É. V. Prozorova, "Similar unsteady boundary layer flows," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1975).
2. É. V. Prozorova, "Some similarity problems in unsteady boundary layer," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1976).
3. G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge Univ. Press (1967).
4. L. G. Loitsyanskii, Laminary Boundary Layer [in Russian], Fizmatgiz, Moscow (1962).
5. J. Fertsiger and G. Caper, Mathematical Theory of Fluid Transfer Processes [Russian translation], Mir, Moscow (1967).
6. V. D. Vilenskii, "Unsteady convective heat transfer in external flow," Teplofiz. Vys. Temp., No. 5 (1974).